

# Models, Model Theory, and Modeling\*

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For some time now, I have been exploring the relations between model theory, logic, and the semantics of natural language. Some might think these are all nearly the same thing. A view might hold that model theory is the fundamental way we understand logic, and that it is equally the basic tool in semantics, and that indeed doing semantics is just the same as doing logic. We can do it for formal languages, or natural ones. This is a fairly strong and contentious set of claims, but forms a natural and I think appealing view. (The latter claims about natural language were at least held by Montague (1970).) The number of books and papers whose titles have phrases like ‘model-theoretic semantics’ or ‘logic and language’ might also make one think this view has some currency. Regardless, in earlier papers (Glanzberg, 2014, 2015) I have urged caution in these matters. I have argued for a real but limited role for model theory in semantics of natural language, and I have likewise argued for limited connections between natural language and logic proper.

In this paper, I shall return to the relations between logic and semantics of natural language. My main goal is to advance a proposal about what that relation is. Logic as used in the study of natural language—an empirical discipline—functions much like specific kinds of scientific models. Particularly, I shall suggest, logics can function like analogical models. More

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provocatively, I shall also suggest they can function like model organisms often do in the biological sciences, providing a kind of controlled environment for observations.

My focus here will be on a wide family of logics that are based on model theory, so in the end, these claims apply equally to model theory itself. I do not think this makes model theory unique. Virtually all the tools of contemporary logic have proved fruitful in studying natural language: model theory, proof theory, recursion theory, set theory, intensional and extensional logics, classical and non-classical ones, . . . But, model theory does offer a particularly important set of applications, so I shall focus on it. At the same time, model theory offers a particular way of understanding what is basic about logic. Along the way towards arguing for my thesis about models in science, I shall also try to clarify the role of model theory in logic. At least, I shall suggest, it can play distinct roles in each domain. It can offer something like scientific models when it comes to empirical applications, while at the same time furthering conceptual analysis of a basic notion of logic.

The plan of the paper is as follows. In section 1, I shall begin with logic itself. I shall argue that model theory, even some of the complex mathematics we find in modern model theory, can help us understand some basic issues about logic. It can even support a broad conceptual analysis of the nature of logic. In section 2, I shall turn to applications to natural language; and briefly, to other aspects of cognition. I shall argue there that model theory provides tools that function like scientific models for the study of empirical phenomena. Finally, in section 3, I shall ask what sorts of scientific models logic and model theory provide. I shall argue that they provide two sorts: they provide both analogical models, and models that function much the way model organisms function in the life sciences.

Before proceeding, let me mention some unfortunate terminological problems. Clearly, the term ‘model’ is being used multiple ways. These uses are well-established, and so it is pointless to try to introduce new terminology. Sometimes I shall have to just let context disambiguate. But for the most part, when I say ‘model’ I mean what model theory—the branch of logic—has in mind. I shall try to say ‘scientific model’ or ‘model in science’ to distinguish such models from model theory. Also, for the most part, I shall discuss a number of logics, and be quite liberal in what I count as a logic. But at some points, I shall ask about a fundamental philosophical issues of what really counts as logic, and which, or how many, of the logics mathematics provides are really logic. I shall write ‘LOGIC’ in capital letters to

mark this philosophically fundamental notion (assuming that indeed it is a coherent and well-defined one). Neither of these notational fixes is elegant, but they will suffice.

## 1 Logic and Model Theory

Before getting to empirical applications, I shall consider some ways of thinking about logic, and perhaps LOGIC.

What is logic? There are many potential answers to this question. One will be especially important to empirical applications, but there are others. Taking inspiration from work of Cook (2002), we can consider several, options:

**The instrumentalist view of logic:** Logic is just mathematics. Like any mathematical machinery, you can use it however you like. It can be used in an instrumentalist fashion, to roughly track some phenomena, perhaps in language or reasoning, but offering no real explanations of the phenomena in question. Any implications from the mathematical machinery can be viewed as convenient fictions.

**Logic as description:** Logic describe what is really going on with the truth conditions, consequence relations, etc. of various discourses.

**Logic as conceptual analysis:** There are fundamental facts about consequence and other logical properties. These facts are revealed by conceptual analysis, and logic provides that analysis. Pluralists might think there are many such sets of facts, singularists will hold there is only one.

**Logic as modeling:** Logic can provide models (in the sense of scientific models) that help us understand various phenomena, especially in language and reasoning. It can be a fruitful ways to represent and study these phenomena. But it is one tool among many, and there are many ways phenomena can be modeled, with different benefits and limits.<sup>1</sup>

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<sup>1</sup>Cooks builds on work of Shapiro (1998), who in turn notes a suggestion of Hodes (1984). I have modified Cook's presentation in several ways, but kept to the spirit of his taxonomy. I added the conceptual analysis option. My statement of the instrumentalist view is somewhat broader than Cook's. He has in mind a fairly specific set of applica-

As Cook notes, these positions are extreme, and it is not clear if anyone has held them in the forms stated. But even if this range of options is something of a caricature, it offers a useful structure within which to ask very general questions about logic.

If we assume there is such a thing as LOGIC—a philosophically fundamental notion—then it seems to go most naturally with the Logic as conceptual analysis option, though perhaps some empiricists might opt for the Logic as description option. I shall argue that in the end, the Logic as modeling option is the best one for thinking about empirical applications of logic.

With these options in mind, I shall turn to the question of the relation of model theory to logic. Model theory as it is done these days includes a lot of advanced mathematics. It is usually done as a branch of abstract mathematics, typically within set theory, but with many applications, to algebra, geometry, and so on. Especially if we are thinking of LOGIC, we might worry that such mathematics and LOGIC have little to do with each other. The point is made vividly by Sacks (1972, p. 1):<sup>2</sup>

Part of the blame belongs to B. Dreben who once asked with characteristic sweetness: “Does model theory have anything to do with logic?” It is true that model theory bears a disheartening resemblance to set theory, a fascinating branch of mathematics with little to say about fundamental logical questions . . .

In contrast, many philosophical logicians find model theory to be essential to logic, or at least to LOGIC. The point is made vividly by a wonderful quote from Routley & Meyer (1973, p. 199) via Restall (2000):

Yea, every year or so Anderson and Belnap turned out a new logic, and they did call it  $E$ , or  $R$ , or  $E_{\bar{I}}$  or  $P - W$ , and they beheld each such logic, and they were called relevant. And these logics were looked upon with favor by many, for they captureth the intuitions, but by many they were scorned, in that they hadeth no semantics.

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tions to vagueness (which is the main focus of his paper), while I have a wider range of applications in mind. One might worry that not any old piece of mathematics is *logic*. I agree, but given the range of different mathematical techniques that do seem to count, I am avoiding that demarcation issue here.

<sup>2</sup>This is from the introduction to Sacks’ book *Saturated Model Theory*, which covers a great deal of the mathematics of model theory as it was done at the time.

(That is, until Urquhart (1972) provided one.)

I shall assume throughout that giving a semantics for a logic is an exercise in model theory. Hence, the two views could not be more different. On the one hand, we question of whether model theory has anything to do with logic; on the other, we insist that without a model theory a logic is somehow not good enough.

If we are instrumentalists about logic, of course, there is no issue here. Any mathematics is fine. Model theory is, so is the proof theory that framed many relevance logics, and so on. If we think of logic as description, we might worry more, about whether various pieces of mathematics help with such descriptions. But the question is most vivid for the Logic as conceptual analysis view. There, the questions of whether one must have a model theory as part of one's conceptual analysis; and if so, what role such mathematics could play in any conceptual analysis seem urgent.

I shall propose a way of bridging these two opposing views, that will allow us to see sophisticated mathematics as playing a role in conceptual analysis, at least a background role. This will show that instrumentalist and conceptual analysis views of logic can work together.

The key observation is one that Sacks (1972) already made. The quotation above continues:<sup>3</sup>

But the resemblance is more of manners than of ideas, because the central notions of model theory are absolute, and absoluteness, unlike cardinality, is a logical concept. That is why model theory does not founder on that rock of undecidability, the generalized continuum hypothesis, and why the Los conjecture is decidable. . . Los conjectured and Morley proved that if a countable theory is  $\kappa$ -categorical for some uncountable  $\kappa$ , then it is  $\kappa$ -categorical every uncountable  $\kappa$ . The property " $T$  is  $\kappa$ -categorical for every uncountable  $\kappa$ " is of course an absolute property of  $T$ .  
(Sacks, 1972, pp. 1–2)

Model theory has properties that do lend themselves to the study of fundamental logical notions.

Sacks takes an abstract, and somewhat technical, view of this issue, but the point is quite general. Many model-theoretic notions, starting with model-theoretically defined consequence relations, definability properties, many

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<sup>3</sup>This passage is also discussed by Kennedy (2015).

model-theoretic properties of theories, and so on, are relevant to how we understand fundamental aspects of logic, and perhaps even LOGIC. And indeed, the way we can find absolute properties in model theory is a good clue to this fact.

To fix ideas, let us start with a familiar way of thinking about logic that supports a conceptual analysis view. We take the most fundamental aspect of logic to be logical consequence, and take the model-theoretic approach to logical consequence pioneered by Tarski (1935, 1986) to be the core of a conceptual analysis of logic.

This is sometimes called the ‘semantic conception of logical consequence’, and it embodies a long-standing tradition of thinking about the nature of LOGIC (the philosophically fundamental notion), and is well-viewed as a kind of conceptual analysis of LOGIC. I’ll briefly review some of the key ideas of this view.

LOGIC, according to this view, describes something fundamental: valid arguments, which are a fundamental constraint on good reasoning. The tradition from Tarski (1936) places two constraints on LOGIC: *necessity* and *formality* (cf. Beall & Restall, 2009; Etchemendy, 1990; Sher, 1991):

- Necessity: If  $S$  is a consequence of a set  $X$  of sentences, then the truth of the members of  $X$  necessitates the truth of  $S$ .
- Formality: Logical consequence holds ‘in virtue of the forms’ of sentences.

Note, that even before spelling these out further, they are enough to distinguish logical consequence from e.g. inductive support.

Articulating these requirements more fully can be done via the notion of a ‘case’ from Beall & Restall (2006). Providing a logic can be done by providing a range of cases of some sort. The range must be sufficient to capture necessity: preserving truth in all cases must suffice to establish necessity. Cases also need to support formality, by allowing fixed treatment for logical constants. Validity of arguments is characterized by what Beall & Restall (2006) call the *generalized Tarski Thesis*:

GENERALIZED TARSKI THESIS (GTT): An argument is valid <sub>$x$</sub>  if and only if, in every case <sub>$x$</sub>  in which the premises are true, so is the conclusion.

Beall and Restall are logical pluralists, and so they subscript  $\text{valid}_x$  and  $\text{case}_x$  to allow many different instances. But regardless, what we have here is a fairly common conceptual analysis of logical consequence. This can be seen as a conceptual analysis of LOGIC. It is one of many contenders, but will serve our purposes here well as an illustration of a conceptual analysis.

Model theory enters to tell us what cases are. In the tradition of classical logic, cases are just models in the usual sense from model theory. Beall and Restall also consider possible worlds, situations, and so on, to give a wider range of logics. But model theory has the mathematical wherewithal to capture any of these.

So far, it looks like model theory only relates to the conceptual analysis of logic, or to LOGIC, in a very modest way. It tells us what cases are, but that barely scratches the surface of the rich mathematics of model theory.

But this is to underestimate the complexity of the mathematics of these ‘cases’, which is of course model theory. Even if we fix the standard classical notion of a model, there is a huge range of different ways we can use them as cases, and they produce different logics. This is the family of *model-theoretic logics* as explored e.g. by Barwise & Feferman (1985). Examples include:

- Classical first-order logic.
- Second-order logics of various strengths.
- Logics of generalized quantifiers  $\mathcal{L}(Q)$  for various families of quantifiers  $Q$ .
- Smaller infinitary logics like  $\mathcal{L}_{\infty,\omega}$  (with arbitrary conjunctions and disjunctions but finitely many quantifiers in any sentence), and its fragments.
- $\mathcal{L}_{\kappa,\lambda}$  (with  $< \kappa$ -sized conjunctions and disjunctions, and  $< \lambda$ -many quantifiers in any sentence).

We can study the properties of these in depth, including forms of compactness, interpolation, etc. We can sometimes, as with Lindström’s theorem (Lindström, 1969), characterize these logics in substantial ways. As Sacks notes, we find reasonable degrees of absoluteness in many of these cases (though not all!). We can, of course, find even more options if we depart from classical models, including many intensional, relevance, and other logics.

We use substantial mathematical resources—substantial parts of the mathematical subject of model theory—to formulate and understand these options. So sophisticated mathematics can play a role in our conceptual analysis. It can help us understand a wide range of options for what formally fits with our conceptual analysis. Of course, this a number of further questions for our conceptual analysis. Which of these are LOGIC, if there is such a thing? Can one or many be LOGIC? Are there important properties that help us decide? I suspect there are. Compactness properties are a good example. Though I am not sure that every aspect of model theory plays a role here (would the applications to geometry?), we do see that substantial mathematics interacts fruitfully with conceptual analysis. Model theory does have much to do with logic understood as conceptual analysis, and maybe even has something to do with LOGIC.

We see here ways that two approaches to logic, the Logic as instrumental and the Logic as conceptual analysis approaches, can fruitfully interact. Perhaps much model theory in mathematics is done simply as pure mathematics, with little more than the instrumentalist view in mind (what else would mathematicians do than mathematics?). But mathematics can be applied in many ways, and it turns out that model theory has interesting, and I think rich, applications to Logic as conceptual analysis.

The application can be seen as follows. The mathematics of model theory give us many candidate logics. At least to some extent, they meet the conditions conceptual analysis provides. So, they seem to be candidates for being LOGIC, and we then have to explore, sometimes with real mathematical sophistication, which candidates are really the right ones.<sup>4</sup>

Let us call the kinds of logics that we generate this way model-theoretic logics.<sup>5</sup> Model-theoretic logics have a home in pure mathematics, which perhaps may suggest an instrumental view, but they can be part of a conceptual analysis view as well.

With that, let us ask the next question: what does model theory have to do with natural language or other empirical phenomena?

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<sup>4</sup>Not every exercise in the mathematics of building models seems to offer the kinds of conditions our conceptual analysis requires. I am not sure that some models of Lambek calculi or Linear logics do. But I shall not argue that here.

<sup>5</sup>This includes the sorts of logics called model-theoretic by Barwise & Feferman (1985), but is wider, as it also includes intensional logics and many others.



## 2 Relations to Language (and Cognition)

So far we have focused on foundational or conceptual matter: what is logic, or even LOGIC, and what role does model theory play in it. I argued that we can view model theory as having instrumental roots, but that it supports conceptual analysis views of logic as well. Indeed, these come together in an interesting way. But I did not consider either Logic as description or Logic as modeling approaches. Logic as modeling does not seem to have any place when we address only such conceptual questions. There is no empirical phenomenon to model. But I shall argue, it is the best option when we turn to empirical phenomena. I have already argued in effect that Logic as description is a mistake in other work, that I shall review here. My main thesis for this section is that when we come to empirical applications, we should rely on Logic as modeling exclusively.<sup>6</sup>

Let me begin with the Logic as description view. It presupposes that natural languages, or discourses formed in them, have a logic to begin with. This is what in earlier work (Glanzberg, 2015, p. 75) I called the *Logic in natural language thesis*:

A natural language, as a structure with a syntax and a semantics, thereby has a logical consequence relation.

I take the Logic as description view to be stronger than the Logic in natural language thesis. The latter says we can find a logic in natural language. I take the former to hold that it is THE logic, or perhaps LOGIC.

The Logic in natural language thesis is in effect endorsed by Montague (1970, p. 222):

There is in my opinion no important theoretical difference between natural languages and the artificial languages of logicians; indeed, I consider it possible to comprehend the syntax and semantics of both kinds of languages within a single natural and mathematically precise theory.

I am not sure if Montague was assuming a broader descriptivist, instrumentalist, or conceptual analysis view of his ‘single natural and mathematically precise theory’, but descriptivists should also endorse Montague’s claim.

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<sup>6</sup>I suspect most researchers working in empirical areas will not be surprised by such a conclusion. Scientific models are common tools for them, and being told that logic is one may not be a surprise. But among philosophical logicians, the Logic as modeling idea from Cook (2002) is indeed surprising and controversial.

One of the main claims in my (Glanzberg, 2015) was that the Logic in natural language thesis is false. If so, this rules out descriptive approaches to the application of logic to natural language. I argued there that what we find in natural language is not really logical consequence. Descriptively, natural language does not hand us a logic. If this is correct, Logic as modeling is our only option for these applications. But moreover, I shall argue here that the way of thinking about the relation of logic to language I outlined in that paper really is an instance of logic as modeling.

I argued for the claim that the Logic in natural language thesis is false at (perhaps excessive) length already, and I shall not try to repeat the full arguments here. But to get to the new points I wish to make, it will be helpful to make reference to the specific arguments from my earlier work. I presupposed that the conditions of formality and necessity discussed above are constraints on what counts as logic (and so perhaps engaged in some conceptual analysis). With that, I gave three distinct arguments.

First was the argument from absolute semantics. As noted by Davidson (1967) and Lepore (1983), semantics for natural language must be absolute, in that to give correct and non-vacuous truth conditions for sentences we must fix the correct reference and satisfaction conditions for their constituents. This leaves no use for a space of models. We only need real-world reference and satisfaction. For instance, recall that an atomic sentence  $Fa$  will be assigned arbitrary extensions for  $F$  and  $a$  across models. That does not give any substantial truth conditions. If you want to give the truth conditions of *Sam is happy* you need the real-world reference of *Sam* and the real-world extension of *happy*. Hence, what natural language gives us does not satisfy necessity, and we find no genuine consequence relation.

Second was the argument from lexical entailments. I argued lexical entailments will not count as logical consequences (as they either fail necessity or formality).

Finally, I offered the argument from logical constants. I argued that there is no linguistically distinguishing marks of logical constants.

All this made me claim that we do not find any genuine consequence relations in natural language, and so, we cannot take any plausible consequence relation to be the one of natural language. Nor can we take the semantics of natural language to be the semantics that gives any such consequence relation. The Logic in natural language thesis must be false, and if we assume some minimal constraints on what counts as logic, so must any descriptive approach.

This was never to say that we cannot find useful clues to logics within our languages. Historically, we have. But we need to do more than just describe the semantics of our languages to find them. In Glanzberg (2015) I argued that we need to go through a substantial three-fold process to get from a language to a logic. It includes abstraction from absolute semantics to get an appropriate domain of models or cases. In many situations, we do this in a specific way. Absolute semantics provides specific meanings to non-logical expressions. Abstraction allows these to vary, and as has been much discussed, we also usually allow the domain of quantification to vary. We also have to identify the logical constants. Natural language fails to do this, but we must. To achieve formality, we will need to hold the meanings of the logical constants fixed in abstraction. Abstraction and identification work together, and doing both typically yields a common post-Tarskian understanding of a model-theoretic consequence relation.

I argued that this is not sufficient. Another step of *idealization*, is needed. Even after we have performed abstraction, we are still going to be stuck with some idiosyncratic and quirky features of natural language grammar, that we will not want to contaminate our logic. Rather, we want our formal languages to display uniform grammatical properties in important logical categories. So, we must idealize these quirks away. Idealization like this is familiar from modeling in science.

What can result from this process of abstraction, identification, and idealization? The familiar history suggests that classical logic seems to be one likely outcome. First, we identify logical constants to be *or*, *and*, *if*, *not*, *every*, *some*. Then, we use set theory to freely vary extensions of non-logical terms and predicates. This is easy and natural. But we also do substantial idealization. For instance, we modestly idealize the meaning of *or*, and more substantially the meaning of *then* to get  $\vee$  and  $\longrightarrow$ . We make structural idealizations as well. For instance, the standard first-order quantifiers depart from the syntax of natural language, and the scope behavior of quantifiers in natural language differs from what we have in logic. Historically, these idealizations were driven by applications to mathematics, as well as reasons of simplicity and uniformity.<sup>7</sup>

We might find more than classical first-order logic. For instance, plural constructions might lead you to second-order quantifiers (Boolos, 1975; Linnebo, 2003; Rayo, 2002; Uzquiano, 2003). The structure of quantified noun

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<sup>7</sup>For a general historical overview of the emergence of first-order logic, see Ewald (2019).

phrases might lead you to  $\mathcal{L}(Q_{(1,1)})$ , the logic of (conservative) binary quantifiers (Barwise & Cooper, 1981; Keenan & Stavi, 1986). Modals, conditionals, and tenses can get you to intensional logics. We might also be led to depart from classical logic in some ways. Presupposition can get you to  $K_3$  or even  $LP$  (e.g. Beall & van Fraassen, 2003). Many model-theoretic logics are potential results of the process. Perhaps there are good reasons to rule some of these examples in or out, but we have prima facie reasons to consider all of them, if not more. This does not mean any logic will arise by this process directly. It is hard to see infinitary logics resulting from the process. Rather, they seem to arise as generalizations once we have a logic in hand.

Now, we can return to logic as modeling. We can observe that any instance of this process describes a modeling exercise, and the results are best understood as certain kinds of scientific models. As I mentioned, idealization is a familiar aspect of modeling. Abstraction is a form of generalizing, which is also an aspect of modeling. Identification is too. You identify things you are modeling and things you are not, in any instance of modeling. Without those steps, we do not get a recognizable logic, so getting logic in the way I described should best be thought of as an exercise in a certain kind of modeling.

Though I have focused on language, it is worth noting briefly that we might say similar things about the role of logic in human reasoning. A good example is the PSYCOP model from Rips (1994). This is a model, based on a specific (classical) natural deduction system, with a generous rule set. It is a memory-based model, as it models reasoning as constructing proofs in working memory. It is part of a larger model, with goal and control systems. In this case, not surprisingly, Rips extensively tested his model against subjects judgements of validity for arguments, judgements of validity of proof steps under timed conditions, and tested failures of reliability for conditionals and negation, and so on. Logics can provide scientific models in many forms, applying to language and other aspects of cognition.

So, I claim, when logic is applied to language and cognition, we should adopt the Logic as modeling approach. We find logics for natural language by a process of scientific modeling. But if so, what kinds of models do we get, and what do they tell us about the phenomena? And, returning to one of our main themes, what is the role of model theory in this kind of scientific modeling?

One role model theory plays in Logic as modeling is just the same one it played when we considered conceptual analysis. Model theory can provide us

a broad and rich range of model-theoretic logics. These are candidate models, and understanding them will help us with our modeling exercise. Indeed, the process I described above is one tailored to produce model-theoretic logics, and the mathematics of model theory can help us do that, and understand the results. So, we can see model theory as providing the mathematical resources that give us certain kinds of scientific model, and allow us to understand how they work. Those kinds of models are just the kinds we get by the process I described above.

The question then becomes, what value do these sorts of models have? Why model that way? One answer is not surprising. These models help us understand the behavior of expressions close to logical constants, of course.

Here is one example among many. Generalized quantifier logics, with their mathematically rich model theory, offer insights into the properties of natural language quantificational determiners: expressions like *most* (in languages that have them<sup>8</sup>).

Here is a logical property expressible in a logic with generalized quantifiers. We look at  $Q_M$ , the extension of a generalized quantifier in a model with domain  $M$ . This will be essentially a set of sets. We can then define such properties as

- (1) EXT for type (1) quantifiers: For any  $A \subseteq M \subseteq M'$ , we have  $Q_M(A) \longleftrightarrow Q_{M'}(A)$ .

EXT is a strong form of domain restriction. It is not something we would find if we just specified the absolute truth conditions for sentences, as it requires varying domains for each quantifier. We have to engage in the kind of scientific modeling I just described to find this property. And yet, its analog for natural language determiners seems to hold. They are highly domain-restricted, in much the way this property describes. A classic theorem tells us that EXT plus permutation invariance implies isomorphism invariance (EXT + PERM implies ISOM). Natural language quantifiers also show strong permutation invariance. So, we have learned something about our quantifiers, using our logic, i.e. our scientific model.<sup>9</sup>

We can keep adding to this list of examples. The entailment properties of quantifiers has proved useful in understanding the surprising behavior of ex-

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<sup>8</sup>It is unlikely that all languages do. See Bach et al. (1995) and Keenan & Paperno (2012)

<sup>9</sup>See Peters & Westerståhl (2006) for more details and references. I have discussed this kind of use of model theory more in my (Glanzberg, 2014).

pressions like *any*: so called negative polarity items.<sup>10</sup> Looking to intensional systems, we can say the same about modals, conditionals, tenses, and so on. Generally, we can fruitfully study the properties of logically rich expressions using the tools of model theory and model-theoretic logic as scientific models, in much the way I described above. I noted above that absolute semantics show us that building these models is not the immediate job of specifying the truth conditions of sentences and their parts, and so is not the most immediate task for semantics. But it can be useful nonetheless.

This particular sort of scientific model has its limits. That is not a surprise. Any model has limits. Let me mention one. I am skeptical of how much model-theoretic logics will help with lexical entailments, which is an important and data-rich area of semantics.<sup>11</sup> We can indeed, as Carnap (1952) and Montague (1973) both noted, sometimes capture lexical entailments as constraints on spaces of models. These appear as meaning postulates, like the familiar:

$$(2) \quad \forall x(Bachelor(x) \longleftrightarrow [Human(x) \wedge Male(x) \wedge Adult(x)])$$

The result is a step to partially interpreted languages but their model theory is also a rich subject (cf. Barwise, 1975).

Since work of Zimmermann (1999), many linguists have been cautious about meaning postulates, and tend to prefer to re-write these rules as lexical decomposition rules. More importantly, many cases of lexical entailment do not indicate this kind of restriction on a space of models.

A good example is the dative alternation and its puzzles.

- (3) a. Anne gave Beth the car. (Double Object (DO))  
 b. Anne gave the car to Beth. (Prepositional Object (PO))

These are near synonyms. But not quite:

- (4) a. Anne sent a package to London.  
 b. # Anne sent London a package.

We also see different entailments:

- (5) a. Mary taught John linguistics.  
       ENTAILS  
       John learned linguistics.

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<sup>10</sup>See Giannakidou (2011) for an overview.

<sup>11</sup>For an interesting discussion of what meanings can be captured with model-theoretic tools, see Sagi (2018).

- b. Mary taught linguistics to John.  
DOES NOT ENTAIL  
John learned linguistics.

We have a complicated set of patterns here!

How should we understand them? This remains controversial. But let me illustrate briefly with one idea. We should look for a rich lexical decomposition that tracks some conceptual difference that this pattern displays. One approach posits a polysemy between the DO and PO cases (Krifka, 1999; Pinker, 1989):<sup>12</sup>

- (6) a. DO: NP<sub>1</sub> CAUSES NP<sub>2</sub> TO HAVE NP<sub>3</sub>
- b. PO: NP<sub>1</sub> CAUSES NP<sub>2</sub> TO GO TO NP<sub>3</sub>

These frames show different sorts of meaning: a meaning involving causing to have, versus a meaning involving causing to go to. The two different meanings are realized in English with different syntax.

One virtue of this proposal is that it helps explain why we get *# London a package* above. This is a DO form, and so requires HAVE. London does not have a package. It also explains the entailment we noted by assuming HAVE typically includes mastery. ‘Successful transfer’ is required for HAVE.

So, perhaps this is a good explanation. As I said, it remains controversial. But it is a plausible exercise in the kind of scientific modeling we might do for natural language. We have a complex phenomenon, and we build an abstract model that helps explain what is happening.

But this is not an exercise in logic, or scientific model-building with model-theoretic logics. Rather, it is an analysis of the components of a verb frame, which tells us how a class of verbs behave. Perhaps we could do some logic to understand the components like CAUSE or DO, or with locations, GO. But that would not give us the insight this theory offers. It offers a different kind of analysis, not one from logic-like modeling.

So, even when looking at entailments in language, logic as modeling may not be the most useful approach. Like all scientific models, this sort has its limits. Model theory and model theoretic logics offer interesting abstract scientific models of some entailment patterns. These have proved useful for studying some ‘logical’, mainly functional, expressions. But they do not seem

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<sup>12</sup>For a critical discussion both of the data and the proposal, see Rappaport Hovav & Levin (2008).

of much help for difficult problems like the lexical entailment I illustrated above,

But this raises a further question. When we can use models to model, what sorts of models do they provide? And how does they really help us to learn about phenomena?

### 3 Learning from Models

I have argued that when applied to empirical phenomena like natural language and human reasoning, we can usefully see logics, especially model-theoretic logics, as offering scientific models. But there are many sorts of models in science, and we do different things with them and learn from them differently. I shall conclude by suggesting that logics can work as models of at least two sorts. They can function as what are called analogical models, but also in ways similar to model organisms.

The way I described generating a logic starting with a natural language puts weight on idealization, along with identification and abstraction. This suggests that perhaps logics function as what are sometimes called idealized models in science.<sup>13</sup> The standard example is the frictionless plane, which idealizes away friction. Notably, we have surfaces of varying amounts of friction, and this idealized model simply pushes that to an idealized limit. As I recall from my college physics class, we use this model to help us work out basic laws of mechanics.

To some extent, logics can behave like this. A logic of generalized quantifiers idealizes away various features of natural language, including syntax and scope restrictions on quantifiers, to provide a general picture of what scope and quantifier meanings look like. We use this to understand the phenomena in question.<sup>14</sup>

But this is not quite right. After all, we do not stare at logics of generalized quantifiers and figure out the laws of scope in natural language. We cannot, as these logics have idealized away what we would need to do so.

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<sup>13</sup>I am not a philosopher of science, and am very much working as an amateur when it comes to scientific models. As amateurs sometimes do, I am trying to just follow the experts, in this case with the help of the *Stanford Encyclopedia of Philosophy* entry on models in science by Frigg & Hartmann (2009).

<sup>14</sup>For an extensive overview of scope phenomena in natural language, see Szabolcsi (2012).



A better picture comes from what are often called analogical models in science. The billiard ball model of gases is a standard example. These models analogically exploit similarities between systems. So, a system of colliding billiard balls is analogically related to a gas. These models also help us to find basic, sometimes idealized laws. But they are not simply limits of what is real. Gases are not made of billiard balls, even in a limit.

I think we rely on such analogical roles when we use logic to model natural language. Take quantifier scope, a messy phenomenon in natural language. We understand it best after we abstract away to a logic with quantifiers. We then look back and say that scope in natural language must be something like we see there. Not exactly, but by analogy.

My suspicion is that many applications of model theory to natural language and cognition are like this. Model-theoretic logics give us idealized and often analogical models of various aspects of language, or perhaps cognition. We can use them to try to formulate more accurate hypotheses about how language really works, and can learn from general comparisons or analogies. Note, for instance, that the idea that possible worlds are like models is already a clear kind of analogical modeling.

Analogical models are often useful to get inquiry going. But they have limits, of course. One is that they do not, typically, have parameters we can manipulate. Many models are systems of equations, or computer simulations, that have such manipulable parameters. (Basically, constants that can be adjusted to affect the behavior the model.) This is important, as it allows us to test models against data, and refine them by adjusting parameters. We can then use the models to generate new predictions, which are again tested against data. This cycle allows the building of more refined models, which help us improve our understanding of some phenomenon and make better predictions.

No doubt in some cases, logics can be used like this. A clear example is the PSYCOP model of Rips (1994), which was built to be just such a model. Perhaps there are others. We might think of the accessibility relation of a Kripke model as a kind of manipulable parameter, or perhaps the strength of a logic of generalized quantifiers. Perhaps this is right in some cases, but in many, I find the comparison strained. It is not clear that we simply adjust the accessibility relation as a parameter, with a set range of values, in response to data. We certainly cannot do that with our logics of generalized quantifiers. At the same time, I think there is a better way to see how these sorts of tools function as models beyond their analogical roles.

The range of what counts as models is very large, and I shall leave it to the philosophers of science to decide whether it forms a homogenous class or kind. But among what are called models are model organisms, often worms or mice. I shall suggest that logics often function more like model organisms than other sorts of models.

Model organisms are often carefully designed, by breeding or genetic engineering. For instance, if you want to study a human disease, you might design a model organism to simulate relevant aspects of the human. You can then observe the phenomenon in a controlled environment, typically by infecting your carefully designed mouse and seeing what happens.<sup>15</sup>

Animal models can be changed in the face of data, but in many cases, they do not have straightforward manipulable parameters. Certainly, when they are designed, allowing fairly straightforward changes is a valuable feature. But often when an animal model fails, it is a significant task to build an improved one.

I think in many applications, logics function surprisingly like animal models. Take the case of a generalized quantifiers again. A logic of generalized quantifiers in part, but only in part, reflects the way quantifiers work in natural language. The same is true of animal models. We can use the logic as a controlled environment to study the behavior of quantifiers. Drop a quantifier into it, and we can see a set of entailment patterns, and a number of other logical properties, like monotonicity properties, definability properties, EXT mentioned above, and so on. We can watch these behaviors, and see how that compares to quantifier behavior in natural language. We can generate predictions from the model. Sometimes, we can change the model itself. We might restrict ourselves to conservative quantifiers, for instance. We can sometimes even find a manipulable parameter. Restricting ourselves to finite models can provide interesting results, especially about discourse processing. But then, the size of a domain can be manipulated.<sup>16</sup>

In practice, I suggest, we use logics to study natural language both like analogical models, and like model organisms. We can use lots of other tools, and we do. I have focused on model-theoretic logics, but proof theory has many applications. So do non-logical methods. Decision theory comes to mind. So do theories of concepts and categorization. We have lots of

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<sup>15</sup>This is a common technique among biomedical researchers. For an overview, see for instance Fox et al. (2007).

<sup>16</sup>See, for instance, results from Keenan & Stavi (1986).

tools, but model-theoretic logics are among them, and often used as model-theoretic logics to provide scientific models in these two senses.

I have illustrated specific roles for model theory in both foundational approaches to logic and to the study of natural language. In both, the rich mathematics of model theory can be important. I doubt these are the only roles for model theory in applications, but I think they are central ones. Model theory provides us a rich stock of model-theoretic logics. In foundational studies, these can be important options for conceptual analysis, and raise a number of foundational questions about the nature and scope of logic. Perhaps this might help us to understand the nature of LOGIC. When we turn to applications to empirical matters like language and cognition, these logics stand in the relation of models in science to phenomena. Specifically, I suggest, we use them like analogical models, and sometimes much like model organisms are used. Such models have valuable, but I think limited applications, as all good models do. But generally, when applied to language and cognition, we should think of logic and model theory as much more like models in science.

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